Illustration of the universal multi-exponential decays inequality

$$S(t; w, r) = \sum_{i=1}^{n} w_i \exp(-r_i t) \geq W \exp(-R t)$$

where

$$w \equiv \{w_1, w_2, ..., w_n\}, \quad r \equiv \{r_1, r_2, ..., r_n\}, \quad w_i \geq 0, \quad r_i \geq 0$$

$$W = n\mathbf{G}(w) = n\left[\prod_{i=1}^{n} w_i\right]^{\frac{1}{n}}, \quad R = \mathbf{A}(r) = \frac{1}{n}\left(\sum_{i=1}^{n} r_i\right),$$

G and **A** indicating the Geometric and Arithmetic means, respectively.

Horizontal axes: t , Vertical axes: S(t; *w*, *r*)
For clarity, all possible combinations of linear and logarithmic scales are used.
Apart from the scale types, however, the four graphs are identical.

Numeric values used: *w* = {0.6, 0.3, 0.1}, r = {1.8, 0.6, 0.2}
The bold, black trace is the mono-exponential decay corresponding to the right-hand-side of the inequality. The six colored traces are the S(t; w, r) decays for the six possible permutations of the three r_i values (while keeping w_i's fixed). The fastest-decaying one (brown) corresponds to the ordering shown above. The slowest-decaying one (orange) corresponds to the reverse ordering r = {0.2, 0.6, 1.8}.